# Final Exam

Catalyst 2015

August 29, 2015

**Instructions:**

This exam consists of 5 problems and 2 extra credit challenge problems (denoted as **EXTRA CREDIT**). The test is out of 100 points. Not all problems are equally difficult, so we recommend looking through the exam to get a scope of the questions before you begin. Please show all work. Good luck!

**Name:**

**Problem 1** (15 points)**.** *Consider the function* $f(x) = 3x^2$*. Find* $f'(a)$ *(the derivative at a general point* $x = a$*) using the definition of the derivative as* $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$.

**Problem 2** (15 points; 7.5 points per part). *Let* $\overrightarrow{a} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ *and* $\overrightarrow{b} = \begin{pmatrix} 5 \\ 8 \\ 10 \end{pmatrix}$.

(a) *Rewrite these vectors as linear combinations of* $\hat{\imath}$, $\hat{\jmath}$ *and* $\hat{k}$.

(b) *Calculate* $2\overrightarrow{a} + \overrightarrow{b}$.

**Problem 3** (30 points; 5 points per part). *Let* $f(x, y) = z = 3x^2 + y^2 + 4$.

(a) *Using the definition of the partial derivative, compute the following:*

$$\frac{\partial f}{\partial x}(1, 3)$$

(b) *Using the definition of the partial derivative, compute the following:*

$$\frac{\partial f}{\partial y}(1, 3)$$

(c) *Note that:*

$$f(1, 3) = 16$$

$$f(1.1, 3) = 16.63$$

$$f(1, 3.1) = 16.61$$

$$f(1.1, 3.1) = 17.24$$

*Using the appropriate values of* $f$ *given above (not necessarily all are useful) and the fact that the derivative can be approximated as the average rate of change in a small interval, check that your answers to parts* (a) *and* (b) *make sense.*

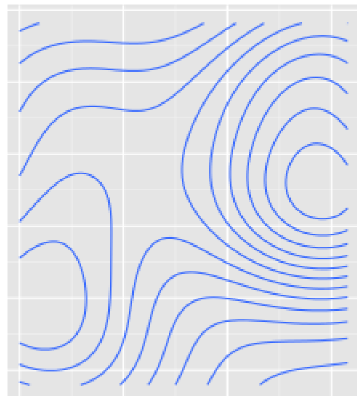
(e) *Find the gradient vector at the point (1,3), namely* $\nabla f(1, 3)$.

(f) *Sketch the vector* $\nabla f(1, 3)$ *in the 2D xy-plane, putting the tail at the origin of your graph.*

(g) *Find the magnitude of* $\nabla f(1, 3)$ *- in other words, compute* $\|\nabla f(1, 3)\|$. *(It is not necessary to simplify your answer.)*

(Work space for Problem 3 continued)

**Problem 4** (15 points; 7.5 points per part). (a) *Consider the contour plot below. Circle the region where the slope is the largest.*



(b) *In 3D space, if we are given a point, can we find the slope? If we need more information to calculate the slope, what is the missing information?*

**Problem 5** (25 points; 5 points per part)**.** *Let $c$ be a number (scalar) and let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\hat{u}$ be vectors. For the following quantities, state if the quantity is a vector or a scalar (you do not need to show work here). It may be useful to recall that $\overrightarrow{a} \cdot \overrightarrow{b} = \|a\|\|b\|\cos\theta$, where $\theta$ is the angle between the two vectors placed tail-to-tail.*

(a) $\|\overrightarrow{a}\|$

(b) $\overrightarrow{a} \cdot \overrightarrow{b}$

(c) $c \cdot \overrightarrow{a} + \overrightarrow{b}$

(d) $\nabla f(a, b)$, *where $f$ is a function*

(e) $\nabla f(a, b) \cdot \hat{u}$

(f) ***EXTRA CREDIT- 2 points****: What is the significance (meaning) of the quantity in* (e)*?*

**Problem 6 (EXTRA CREDIT - 5 points).** *Find two non-zero vectors,* $\overrightarrow{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ *and* $\overrightarrow{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, *in 3D space such that* $\overrightarrow{a} \cdot \overrightarrow{b} = 0$. *(Non-zero means that you cannot set the vector equal to* $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, *the zero vector.).*

**Problem 7 (EXTRA CREDIT-2 points each).** *Say whether the following sets form a basis for three-dimensional space. Provide a short justification for your answers.*

(a) $S = \{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix} \}$

(b) $S = \{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} \}$

(c) $S = \{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \}$

(d) $S = \{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} \}$

(e) $S = \{ \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \}$

(f) $S = \{ \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{3} \\ 0 \\ 0 \end{pmatrix} \}$

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